বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. Honours Examination 2022

(Under CBCS Pattern)

Semester - VI

Subject : MATHEMATICS

Paper : DSE 4 - T

Full Marks : 60

Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

[MATHEMATICAL MODELLING]

1. Answer any ***five*** questions : $2\times5=10$

(a) Find the Laplace transform of the function $F(t-a)$, where $F(t-a)=\begin{cases}0, t\geq a\\0, t<a\end{cases}$.

(b) If $L^{-1}\{f(p)\}=f(t)$, then prove that $L^{-1}\left\{\int_s^{\infty} f(u)\,du\right\}=\frac{F(t)}{t}$.

(c) Verify the initial value theorem for the function $(2t+3)^2$.

(d) Show that although (2, 3, 2) is a feasible solution to the system of equations

$$x_1 + x_2 + 2x_3 = 9$$
$$3x_1 + 2x_2 + 5x_3 = 22$$
$$x_1, x_2, x_3 \geq 0$$

it is not a basic solution.

(e) Use middle-square method to generate 2 random numbers considering the seed x_0 = 1009.

(f) At work station, 5 jobs arrive every minute. The mean time spent on each job in the work station is $\frac{1}{8}$ minute. What is the mean steady state number of jobs in the system?

(g) Cars arrive at a service station according to Poisson's distribution with a mean rate of 5 per hour. The service time per car is exponential with a mean of 10 minutes. What is the average waiting time at steady state in the queue?

(h) Show that $x = 0$ is an ordinary point of the Legendre's differential equation $\left(1-x^2\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n\left(n+1\right)y = 0$; where n is a constant.

2. Answer any ***four*** questions : 5×4=20

(a) Solve the following LPP by simplex method :

Maximize $z = -x_1 + 3x_2 - 2x_3$

Subject to
$$\begin{array}{rcrcrcl} 2x_1 & + & 3x_2 & - & x_3 & \leq & 7 \\ -2x_1 & & & + & 4x_3 & \leq & 12 \\ 8x_1 & - & 4x_2 & + & 3x_3 & \leq & 10 \\ & & x_1, & & x_2, \quad x_3 & \geq & 0 \end{array}$$

(b) Using convolution theorem of Laplace transform deduce the formula

$$\int_0^1 t^{a-1}\left(1-t\right)^{b-1} dt = \frac{\Gamma\left(a\right)\Gamma\left(b\right)}{\Gamma\left(a+b\right)}, a > 0, b > 0$$

(c) Food X contains 6 units of vitamin A per gram and 7 units of vitamin B per gram and costs 12 paise per gram. Food Y contains 8 units of vitamin A per gram, 12 units of vitamin B per gram and costs 20 paise per gram. The daily minimum requirements of vitamin A and B are 100 units and 120 units respectively. Formulate the problem as a LPP model and find the minimum costs of the product mixture.

(d) What do you mean cycling in linear congruence. Use the linear congruence method to generate 20 random numbers using $a = 5$, $b = 3$ and $c = 16$.

(e) Using Laplace transform solve the following initial-value problem

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = te^{-2t}, y(0) = 1, y'(0) = 0$$

(f) Solve $\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2}$ $x > 0, t > 0$ using Laplace transform where $y(0, t) = 1$, $y(x, 0) = 0$.

3. Answer any ***three*** questions : 10×3=30

(a) (i) Discuss the use of Monte Carlo simulation to model a deterministic behavior; the area under a curve.

(ii) Write an algorithm to calculate an approximation to π using Monte Carlo simulation, considering the random number selected inside the quarter circle $Q : x^2 + y^2 = 1,\ x \geq 0,\ y \geq 0$
where Q lies inside the square.
$S : 0 \leq x \leq 1,\ 0 \leq y \leq 1$

Use the equation $\frac{\pi}{4} = \frac{are\,Q}{are\,S}$. 5+5

(b) Consider the following LPP

Maximize $z = 3x_1 + 5x_2$

Subject to
$$\begin{aligned} x_1 & & &\leq 4 \\ 3x_1 &+ 2x_2 & &\leq 18 \\ x_1, & \ x_2 & &\geq 0 \end{aligned}$$

If a new variable x_5 is introduced with cost $c_5 = 3$ and corresponding vector $a_5 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$;
Discuss the effect of adding the new variable and obtain the revised solution if any.

10

(c) (i) Find $L^{-1}\left\{\frac{1}{\sqrt{s}(s-a)}\right\}$.

(ii) Find the solution of the Bessel differential equation of order λ at the neighborhood of $x = 0$. Discuss the case when $\lambda = 0$. 3+7

(d) (i) If $L^{-1}\{F(s)\} = f(t)$ and $L^{-1}\{G(s)\} = g(t)$ then prove that

$$L^{-1}\{F(s)G(s)\} = \int_0^t f(\tau)g(t-\tau)\,d\tau$$

(ii) Using the result of (i), find $L^{-1}\left\{\frac{1}{(s^2+1)(s^2+4s+5)}\right\}$. 5+5

(e) (i) Prove the Final value theorem $\underset{t\to\infty}{Lt}\, f(t) = \underset{s\to 0}{Lt}\, sF(s)$.

(ii) Solve the following LPP by graphical method

Maximize $z = 10x + 35y$

Subject to 4+6

$$\begin{aligned} 8x + 6y &\le 48 \\ 4x + y &\le 20 \\ y &\ge 5 \\ x,\ y &\ge 0 \end{aligned}$$

OR

[DIFFERENTIAL GEOMETRY]

1. Answer any ***five*** questions : 2×5=10

   (a) Define Fundamental Plane.

   (b) Define surface and curvilinear coordinates.

   (c) Find the equation of the tangent plane of $\sigma(r,\theta)=(r\cosh\theta, r\sinh\theta, r^2)$ at (1, 1, 1).

   (d) Find the asymptotic line on the surface $z = y\sin x$.

   (e) Define developable.

   (f) What do you mean by helix?

   (g) Define geodesics.

   (h) Define surface curve.

2. Answer any ***four*** questions : 5×4=20

   (a) Show that the asymptotic lines of the hyperboloid $\vec{r} = a\cos\theta\sec\psi\hat{i} + b\sin\theta\sec\psi\,\hat{j} + c\tan\psi\hat{k}$ are given by $\theta \pm \psi$ = constant.

   (b) Prove that the geodesics on a right circular cylinder are helices.

   (c) State and prove Euler's theorem on normal curvature. 1+4

   (d) Find the first fundamental magnitudes for the curve $\vec{r} = (u\cos v, u\sin v, cv)$.

   (e) Derive tangential and polar developable associated with a space curve.

   (f) Show that asymptotic lines on the Paraboloid $2z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ are $\frac{x}{a} \pm \frac{y}{b} =$ constant.

3. Answer any ***three*** questions : 10×3=30

(a) (i) Show that the curves $u + v$ = constant, are geodesics on a surface with metric $\left(1+u^2\right)du^2 - 2uvdudv + \left(1+v^2\right)dv^2$.

(ii) Derive the partial differential equation of surface theory. 5+5

(b) (i) Prove that the second fundamental form at any point of the surface has the value which equals twice the length of the perpendicular from continuous point to a point on the tangent plane.

(ii) Discusses the geometric significance of the second fundamental form. 6+4

(c) (i) Define torsion and geodesic curvature. Derive analytical representation of geodesic curvature.

(ii) Find the curvature and torsion of the curve

$x = a\left(u - \sin u\right), y = a\left(1 - \cos u\right), z = bu$ 1+1+4+2+2

(d) For the helicoids $z = c\tan^{-1}\frac{y}{x}$, prove that $P_1 = -P_2 = \frac{u^2 + c^2}{c}$, where $u^2 = x^2 + y^2$ and the lines of curvature are given by $d\theta = \pm\frac{du}{\sqrt{u^2 + c^2}}$ and $z = c\theta$. 5+5

(e) (i) State and prove Gauss-Bonnet theorem.

(ii) Find the geodesics on the ellipsoid of the revolution $\frac{x^2 + z^2}{a^2} + \frac{y^2}{b^2} = 1$.

1+4+5

OR

[BIO MATHEMATICS]

1. Answer any ***five*** questions : 2×5=10

(a) The fish growth model by Von Bertalanffy is given by $\frac{dF(t)}{dt} = \alpha F^{\frac{3}{2}}(t) - \beta F(t)$, where $F(t)$ denotes the weight of the fish and α, β are positive constants. Discuss the stability at the equilibrium point.

(b) Define Lyapunov stable and uniformly stable.

(c) Consider a stock of fish that is being harvested at a constant rate $\frac{dN}{dt} = f(N) - h$, where $f(N) = rN\left(1 - \frac{N}{K}\right)$. What is the maximum sustainable yield for sufficiently small harvest levels h?

(d) Discuss the Allee effect given by $\frac{dN}{dt} = N\left[k_0 - l(N-\mu)^2\right], \mu < \sqrt{\frac{k_0}{l}}$ where k_0 and μ are positive constants.

(e) Define the term immigration and emigration.

(f) What are the physical significance of the dominant eigen value of the Leslie matrix?

(g) What is the saddle-node bifurcation of the system $\frac{dy}{dx} = f(x, \mu)$?

(h) Write short note on Routh-Hurwitz criterion.

2. Answer any ***four*** questions : 5×4=20

(a) Determine the nature of critical point (0, 0) of the system $\frac{dx}{dt} = 2x - 7y,\ \frac{dy}{dt} = 3x - 8y$. Also check the stability of the system at critical point.

(b) Write a short note on Nicholson-Bailey host parasite model.

(c) For the system $\frac{dx}{dt}=x-y-x\left(x^2+y^2\right)$, $\frac{dy}{dt}=x+y-y\left(x^2+y^2\right)$, check the existence of a limit cycle.

(d) Discuss about Holling's type functional response.

(e) Using the concept of S-I-R model, formulate a Covid-19 pandemic model, with out considering vaccination. State all the parameters clearly.

(f) Consider the flow of fluid due to pressure gradient in a tube of radius *a* and length *l*. Find the bounds for the velocity distribution.

3. Answer any ***three*** questions : 10×3=30

(a) (i) Formulate differential equation and find steady state solution of SIR (Susceptible-Immigration-Removal) epidemic model.

(ii) Discuss the stability of the steady state solution of that. 5+5

(b) (i) Find out the steady state solution and discuss the stability of the prey-predator model $\frac{dN_1}{dt}=N_1\left(\alpha-\beta N_2\right);\ \alpha>0,\ \beta>0$;

$$\frac{dN_2}{dt}=-N_2\left(\gamma-\delta N_1\right);\gamma>0,\delta>0$$

where N_1 and N_2 respresent the density of prey and density of predator respectively.

(ii) Give the geometrical interpretation of the above prey-predator model. 6+4

(c) The delayed Lotka-Volterra competition system is given by

$$\frac{dx(t)}{dt}=x(t)\left[2-\alpha x(t)-\beta y(t-r)\right];\alpha>0,\ \beta>0$$

$$\frac{dy(t)}{dt}=y(t)\left[2-yx(t-r)-\delta y(t)\right];\gamma>0,\ \delta>0$$

(i) Obtain the steady-state solutions (if exist).

(ii) Investigate the stability of the non-zero steady-states for $\alpha=\delta=2$ and $\beta=\gamma=1$. 4+6

(d) (i) Deduce Fisher's equation for spreading of genes.

(ii) What are the additional restrictions on Fisher's problem for traveling wave solution? 5+5

(e) Let N(t) be the number of tiger population at any time t. The quotient of birth rate and death rate by the population size N are respectively by,

$$\frac{\text{Birthrate}}{\text{N}} = \frac{3}{2} + \frac{1}{1000}\text{N} \text{ and } \frac{\text{Deathrate}}{\text{N}} = \frac{1}{2} + \frac{1}{3000}\text{N}.$$

Formulate a model (using differential equation) that describes the growth and regulation of this tiger population. Solve for N(t), assuming N(0)=100 and describe the long term behavior of this tiger population as $t \to \infty$. 3+4+3