



বিদ্যাসাগর বিশ্ববিদ্যালয়

**VIDYASAGAR UNIVERSITY**

**B.Sc. Honours Examination 2021**

**(CBCS)**

**1st Semester**

**MATHEMATICS**

**PAPER—C1T**

**CALCULUS , GEOMETRY AND DIFFERENTIAL EQUATION**

*Full Marks : 60*

*Time : 3 Hours*

*The figures in the right-hand margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

Answer any four questions.

4×12

1. (a) Find the equation of the asymptotes of the curve

$$r^n f_n(\theta) + r^{n-1} f_{n-1}(\theta) + \dots + f_0(\theta) = 0$$

(b) If  $I_n = \int_0^{\pi/2} \cos^{n-2} x \sin x dx$  show that

$2(n-1) I_n = 1 + (n-2) I_{n-1}$  and hence deduce

$$I_n = \frac{1}{n-1} \quad 5+5+2$$

2. (a) Circles are described on the double ordinates of the parabola  $y^2 = 4ax$  as diameters. Prove that the envelope is the parabola  $y^2 = 4a(x+a)$ .

(b) If  $y = \sin(m \cos^{-1} \sqrt{x})$  then prove that  $\lim_{x \rightarrow 0} \frac{y_{n+1}}{y_n} = \frac{4n^2 - m^2}{4n + 2}$ .

(c) Find a, b, c such that  $\frac{ae^x - b \cos x + ce^{-x}}{x \sin x} \rightarrow 2$  as  $x \rightarrow 0$ . 4+4+4

3. (a) Show that the arc of the upper half of the cardioid  $r = a(1 - \cos \theta)$  is bisected at  $\theta = \frac{2}{3}\pi$ . Find also the perimeter of the curve.

(b) Show that the curve  $re^\theta = a(1 + \theta)$  has no point of inflexion.

(c) Find the asymptotes of the parametric curve  $x = \frac{t^2 + 1}{t^2 - 1}$  and  $y = \frac{t^2}{t - 1}$ .

4. (a) Show that feet of the normals from the point  $(\alpha, \beta, \nu)$  to the ellipsoid

$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$  lie on the intersection of the ellipsoid and the cone

$$\frac{\alpha a^2(b^2 - c^2)}{x} + \frac{\beta b^2(c^2 - a^2)}{y} + \frac{\nu c^2(a^2 - b^2)}{z} = 0.$$

(b) Find the equation of the right circular cylinder whose axis is

$$\frac{x}{1} = \frac{y}{-2} = \frac{z}{2} \text{ and radius is } 2. \quad 7+5$$

5. (a) Prove that  $\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$ .

(b) Two spheres of radii  $r_1$  and  $r_2$  cut orthogonally. Prove that the radius

$$\text{of their common circle is } \frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}.$$

(c) Find the polar equation of the normal to the conic  $\frac{1}{r} = 1 + e \cos \theta, e > 0$ .

2+5+5

6. (a) Find the equation of the generator of the cone  $x^2 + y^2 = z^2$  through the point (3, 4, 5).

(b) Given that the asteroid  $\frac{2}{x^3} + \frac{2}{y^3} = \frac{2}{c^3}$  is the envelope of the family of

$$\text{ellipses } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ show that } a + b = c.$$

(c) State the existence and uniqueness theorem for the solution of ordinary differential equation. 4+4+4

7. (a) Solve :  $x \frac{dy}{dx} - y = x\sqrt{x^2 + y^2}$ .

(b) If  $m$  and  $n$  are positive integers, show that

$$\int_a^b (x-a)^m (b-x)^n dx = \frac{m!n!}{(m+n+1)!} (b-a)^{m+n+1}$$

(c) Solve  $y = 2px + y^2p^3$  and find the general and singular solutions.

3+4+5

8. (a) Compute the length of the curve  $x = 2\cos\theta, y = \sin 2\theta, 0 \leq \theta \leq \pi$ .

(b) Find the points of inflection on the curve  $r(\theta^2 - 1) = a\theta^2$

(c) If  $I_n = \int_0^1 x^n \tan^{-1} x dx, n$  be a positive integer greater than 2, prove that

$$(n+1)I_n + (n-1)I_{n-2} = \frac{\pi}{2} - \frac{1}{n} \quad 3+3+6$$

Answer any six questions.

6×2

9. Find the value of  $\lim_{x \rightarrow \infty} [a_0 x^m + a_1 x^{m-1} + \dots + a_m]^{1/x}$ , in being a positive integer and  $a_0 \neq 0$ .

10. Let  $I_n = \int_0^1 (\ln x)^n dx$ . Show that  $I_n = (-1)^n \underline{n}$ ,  $n$  being positive integer.

11. The curves  $y = x^n, y^m = x$  ( $m, n > 0$ ) meet at  $(0, 0)$  and  $(1, 1)$ . Find the area between these two curves.

12. Find  $\alpha$  if  $x^\alpha$  be an integrating factor of  $(x - y^2)dx + 2xy dy = 0$ .

**13.** Find the curve for which the curvature is zero at every point and which passes through the point  $(0, 0)$  where  $\frac{dy}{dx} = 3/2$ .

**14.** Solve the differential equation :

$$4x^3ydx + (x^4 + y^4)dy = 0.$$

**15.** Generate a reduction formula for  $\int \tan^n x dx$ ,  $n \in \mathbb{Z}^+$  and  $n > 1$ .

**16.** Find the equations of the straight lines in which the plane

$$2x + y - z = 0 \text{ cuts the cone } 4x^2 - y^2 + 3z^2 = 0.$$

**17.** Find the asymptote (if any) of the curve  $y = a \log \left[ \sec \left( \frac{x}{a} \right) \right]$ .

**18.** On the ellipse  $r(5 - 2\cos\theta) = 21$ , find the point with the greatest radius vector.

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