বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. General Examinations 2022

(Under CBCS Pattern)

Semester - IV

Subject : MATHEMATICS

Paper : DSC 1D/2D/3D - T

Algebra

Full Marks : 60

Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

1. Answer any ***five*** questions : 2×5=10

(a) Find all elements of order 8 in the group $(\mathbb{Z}_{24}, +)$.

(b) If a be a unit in a ring R, show that its multiplicative inverse is unique.

(c) Show that a field contains no divisor of zero.

(d) Determine all distinct left cosets of A_3 in S_3.

(e) Give an example of a group G of four elements e, a, b, c with e as the identity element, where $c^{-1} = c$ but $a^{-1} = b$.

(f) Prove that if n is the order of an element a and P is prime to n, then a^P is also of order n.

P.T.O.

(g) Show that if every element of a group (G, o) is its own inverse, then G is abelian.

(h) A group G is abelian if for all $a, b \in G$, $(ab)^2 = a^2b^2$.

2. Answer any ***four*** questions : 5×4=20

(a) Show that a non trivial finite ring having no divisor of zero is a ring with unity.

(b) If S and T be two subrings of a ring R, then show that $S \cap T$ is a subring of R.

(c) Let M_n be the set of all $n \times n$ nonsingular matrix. Show that M_n forms a group under matrix multiplication. Is this group is commutative? Justify your answer. 4+1=5

(d) State and prove a necessary and sufficient condition of a nonempty subset H of a group $(G, \bullet)$ to be a subgroup of the group $(G, \bullet)$. 1+4=5

(e) Let S be the set of 10th roots of unity. Show that $(S, \bullet)$ is a cyclic group. Find all possible generators. Have these generators any special name? 4+1=5

(f) Let H be a subgroup of a group G and $a, b \in G$. *Prove that* $aH \cap bH = \varphi$ if and only if b not in aH.

3. Answer any ***three*** questions : 10×3=30

(a) (i) If R be a ring such that $a^2 = a$, for all $a \in R$, then prove that

(1) $a + a = 0$, for all $a \in R$,

(2) $a + b = 0 \Rightarrow a = b$

(3) R is a commutative ring.

Show further that the characteristic of R is 2.

(ii) Prove that every field is an integral domain, but the converse is not true.

6+(3+1)

(b) (i) Prove that the symmetric group S_3 is non-commutative.

(ii) Define divisors of zero in a ring R. Show that the cancellation law holds in a ring R if and only if R has no divisor of zero. 4+(1+5)

(c) (i) If $(G, \bullet)$ be a group such that $(a \bullet b)^n = a^n \bullet b^n$, for an inger $n = p, p+1, p+2$, for all $a, b \in G$. Then prove that G is commutative.

(ii) Prove that a subgroup H of a group $(G, \bullet)$ is a normal subgroup if and only if $a \in H$ and $b \in G$ imply that $b \cdot a \cdot b^{-1} \in H$. 4+6

(d) (i) Define Characteristic of a ring.

(ii) Show that Characteristic of an Integral domain is a either zero or prime number.

(iii) Show that a finite integral domain is a field. 2+3+5=10

(e) (i) Show that Z/S is a quotient ring, if $S = \{5n : n \in Z\}$.

(ii) If R be a ring and $f(x)$, $g(x)$ be polynomials in $R[x]$, then deg $(f(x)\, g(x)) \leq$ deg $(f(x))$ + deg $(g(x))$. 4+6