(b) (i) Expand $f(x) = \sin x,\ 0 < x < \pi,$ in a Fourier cosine series.

(ii) Evaluate the value of integration $\int_0^1 x^4 (1-x)^3 \, dx$, using Beta function.

(iii) Write down the two dimensional wave equation in polar coordinates and solve it to find the eigen values and eigen functions in case of a circular membrane. 4+2+4

Total Pages : 4

B.Sc./3rd Sem (H)/PHSH/22(CBCS)

2022

3rd Semester Examination

PHYSICS (Honours)

Paper : C 5-T

[Mathematical Physics - II]

[CBCS]

Full Marks : 40

Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer any *five* of the following : 2×5=10

(a) Define regular and apparent singular point.

(b) What are Dirichlet conditions for a function to be piece-wise regular in a given interval ?

(c) Write down the generating relation of Bessel function and show that $J_{n-1}(x) - J_{n+1}(x) = 2J_n'(x)$.

(d) Define cyclic coordinates. Show that the generalized momentum conjugate to cyclic coordinate is conserved.

(e) Write down the Hermite's polynomial and hence show that $H_n'(x) = 2nH_{n-1}(x)$.

(f) Write down the Laplace's equation in spherical polar coordinates.

(g) Prove that $\Gamma(n+1) = n\Gamma(n),\ n > 0$.

(h) Derive the canonical equations of Hamiltonian.

Group - B

2. Answer any ***four*** of the following : 5×4=20

(a) Express $f(x) = x^2$ as a Fourier's series in the interval $-\pi \le x \le \pi$, hence show that at $x = \pi,\ \sum_{n=0}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. 4+1

(b) Prove that the Legendre's polynomials $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2 - 1)^n$, symbols have their usual meaning. 5

(c) Write down the Euler's equation in variational problems and using this show that the path of shortest (brachistos) time (chronos) of a particle is a *cycloid*. 1+4

(d) Write down the Laplace's equation on a plane in terms of the polar coordinates. Solve it by the method of separation of variables and write the general expression of the solution which is finite at $r = 0$ and single valued in θ. 1+4

(e) A simple pendulum consists of mass m_2, with a mass m_1 at the point of support which can move on a horizontal line in the plane in which m_2 moves. Find the Lagrangian of the system and Lagrange's equations. 3+2

(f) A bar of length L whose entire surface is insulated including its ends at $x = 0$ and $x = L$ has initial temperature $f(x)$. Determine the subsequent temperature of the bar. 5

Group - C

3. Answer any ***one*** of the following : 10×1=10

(a) (i) State Hamilton's principle and derive Lagrange's equation of motion from it. Discuss how the result will be modified for non-conservative forces.

(ii) From the generating function of Legendre's polynomials, show that $nP_n(x) = xP_n'(x) - P_{n-1}'(x)$.

(iii) State the Parseval's identity of Fourier series.

(1+4+1)+2+2